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# Principles for the Adjustment of Centerless Superfinishing Machines

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**Abstract**—Theoretical principles are derived for the adjustment of centerless superfinishing machines, by optimization of the geometry, the kinematic parameters, and forces involved. Models are described, numerical experiments are conducted, and practical recommendations are made regarding the adjustment of the machine tools.

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Superfinishing without centers is characterized by complex physicomechanical and geometric–kinematic aspects of shaping. The traditional approach to machining precision in superfinishing machines does not take full account of the interrelationships between the geometry, the kinematic parameters, and the forces involved [1]. The lack of rigorous methods of calculating these parameters at the design stage and in operation of the equipment considerably diminishes its capabilities.

At present, research on the precision in centerless superfinishing focuses on the design of the shaping system and the optimal adjustment of the machines [2, 3]. However, a correct shaping trajectory does not guarantee the specified manufacturing precision, on account of the action of forces and kinematic aspects. In some cases, these factors lead to nonuniform motion of the blank and hence to nonuniform removal of metal. The need for further precision demands the creation of analytical methods and means of taking account of their mutual influence during adjustment of the equipment.

In centerless superfinishing, the blank is based between two rotating shafts. The contact is closed by force, and the rollers transmit the rotation to the blank by means of the frictional forces. Thanks to skewing of the roller axes and the axis of the blank at an angle $\lambda$, there will be a longitudinal component of the supply velocity along the trajectory of the blank. The rollers are solids of revolution with a complex axial profile, which is in contact with the blank along a spatial line. As a result, the contact angles and frictional conditions vary over the machining length.

The blank's trajectory relative to the abrasive bars determines its longitudinal cross section. Thus, for the shaping of a cylindrical surface, we require a rectilinear trajectory parallel to the oscillation plane of the grinding bars; for a cambered surface, we require an arc trajectory, whose radius matches that of the blank's profile. In centerless superfinishing, the blank's trajectory is specified by a roller system, whose geometric adjustment involves establishing the skew angle $2\lambda$ and interaxial distance $2\nu$ of the two rollers.

A mathematical model for the analysis of the shaping trajectory in centerless superfinishing was first proposed in [3]. It was generalized and further developed in [4]. In that version, the blank's motion is described in the machine tool's Cartesian coordinate system $S_\Sigma$, taking account of its dimensions and position, which is specified by six coordinates (three corresponding to the center and three to rotation around the axes). The shaping system is described by a set of coordinate systems (Fig. 1): $S_\Sigma(X_\Sigma Y_\Sigma Z_\Sigma)$ for the machine tool's frame; $S_{bl}(X_{bl} Y_{bl} Z_{bl})$ for the blank; $S_r^L$ $(X_r^L Y_r^L Z_r^L)$ for the

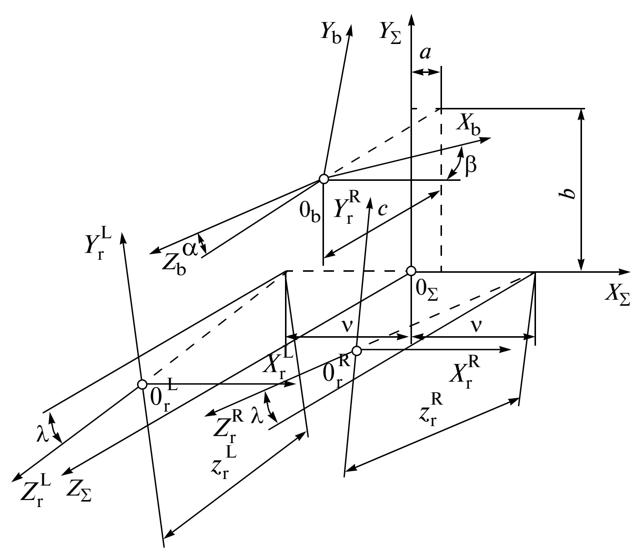


**Fig. 1.** Coordinate systems employed for adjustment of centerless superfinishing machine.

left roller; and $S_r^R$ $(X_r^R Y_r^R Z_r^R)$ for the right roller. Relative to system $S_\Sigma$, coordinate system $S_{bl}$ is rotated by angle $\alpha$ around axis $X_\Sigma$ and displaced by $a$; rotated by angle $\beta$ around axis $Y_\Sigma$ and displaced by $b$; and displaced by $c$ along axis $Z_\Sigma$. Relative to system $S_\Sigma$, coordinate systems $S_r^L$ and $S_r^R$ are displaced by $z_r^L$ and $z_r^R$ along axes $Z_r^L$ and $Z_r^R$, respectively; and rotated respectively counterclockwise and clockwise by angle $\lambda$ around axis $X_\Sigma$ and displaced by $-\nu$ and $\nu$, respectively.

Mutual contact of the surfaces of the blank and the two rollers occurs when the radius vectors $\bar{r}$ and the tangential vectors $\bar{\tau}$ are equal at the contact points. Without loss of generality, determination of the trajectory may be reduced to finding the set of plane cross sections $Z_\Sigma = Z_i$ over the machining length. Formalization of this condition yields the system of equations

$$\left.\begin{aligned}
f_1 &= u^L\sin\psi^L\cos\varphi^L + r\cos\alpha_1\cos\beta \\
&- z_{bl}^L\sin\beta - \nu - a = 0; \\
f_2 &= u^L\sin\psi^L\sin\varphi^L\cos\lambda + A\sin\lambda + r\sin\alpha_1\cos\alpha \\
&+ B\sin\alpha - b = 0; \\
f_3 &= u^L\sin\psi^L\sin\varphi^L\sin\lambda + A\cos\lambda - r\sin\alpha_1\sin\alpha \\
&- B\cos\alpha - c = 0; \\
f_4 &= u^R\sin\psi^R\cos\varphi^R + r\cos\alpha_2\cos\beta \\
&+ z_{bl}^R\sin\beta - \nu + a = 0; \\
f_5 &= u^R\sin\psi^R\sin\varphi^R\sin\lambda - C\sin\lambda + r\sin\alpha_2\cos\alpha \\
&+ D\sin\alpha - b = 0; \\
f_6 &= u^R\sin\psi^R\sin\varphi^R\sin\lambda - C\cos\lambda + r\sin\alpha_2\sin\alpha \\
&+ D\cos\alpha + c = 0; \\
f_7 &= \sin\alpha_1\cos\beta - \cos\psi^L\sin\varphi^L = 0; \\
f_8 &= \cos\alpha_1\cos\alpha + \sin\psi^L\sin\alpha\sin\beta \\
&- \cos\psi^L\cos\varphi^L\cos\lambda + \sin\psi^L\sin\lambda = 0; \\
f_9 &= \cos\alpha_1\sin\alpha - \sin\psi^L\cos\alpha\sin\beta \\
&+ \cos\psi^L\cos\varphi^L\sin\lambda + \sin\psi^L\cos\lambda = 0; \\
f_{10} &= \sin\alpha_2\cos\beta - \cos\psi^R\sin\varphi^R = 0; \\
f_{11} &= \cos\alpha_2\cos\alpha - \sin\psi^R\sin\alpha\sin\beta \\
&- \cos\psi^R\cos\varphi^R\cos\lambda - \sin\psi^R\sin\lambda = 0; \\
f_{12} &= \cos\alpha_2\sin\alpha + \sin\psi^R\cos\alpha\sin\beta \\
&- \cos\psi^R\cos\varphi^R\sin\lambda + \sin\psi^R\cos\lambda = 0,
\end{aligned}\right\} \quad (1)$$

where $r$ is the radius of the blank; $\alpha_1$, $\alpha_2$, $z_{bl}^L$, $z_{bl}^R$ are the angular and linear coordinates of the blank's surface; $R^L$, $R^R$ are the roller radii; $\varphi^L$, $\varphi^R$, $u^L$, $u^R$ are the angular and linear coordinates of the rollers; $\psi^L$, $\psi^R$ are the angles of the roller's generating cones; $2\lambda$ is the skew angle of the roller axes; $2\nu$ is the rollers' interaxial distance; $z_r^L$, $z_r^R$ are coordinates specifying the position of the rollers relative to point $O_\Sigma$; and $A = R^L\cot\psi^L + u^L\cos\psi^L + z_r^L$; $B = r\cos\alpha_1\sin\beta + z_{bl}^L\cos\beta$; $C = R^R\cot\psi^R - u^R\cos\psi^R + z_r^R$; $D = -r\cos\alpha_2\sin\beta + z_{bl}^R\cos\beta$.

We solve Eq. (1) by parametric optimization, with target function

$$\Phi(\alpha_1, \alpha_2, z_{bl}^L, z_{bl}^R, \varphi^L, \varphi^R, u^L, u^R, \alpha, \beta, a, b, c) = \sum_{i=1}^{12} f_i^2 \longrightarrow \min. \quad (2)$$

In solving Eq. (1), we take into account that, if we select a plane cross section ($Z_\Sigma = Z_i$), then the equations $f_5 = 0$ and $f_6 = 0$ become identities (considered separately for each roller and the blank). Hence, we may determine the unknowns $z_r^L$, $z_r^R$, $u^L$, $u^R$ and substitute the results into the equations $f_1 = f_2 = f_3 = f_4 = 0$. This reduces the system in Eq. (1) to ten transcendental equations. The coordinates $a$, $b$, and $c$ determine the position of the point on the blank's axis, relative to which it is rotated by angles $\alpha$ and $\beta$. To determine $a$ and $b$ in the cross sections $Z_\Sigma = Z_i$, we require that $c = Z_i$. As a result, the number of unknowns to be determined by optimization of the target function $\Phi$ is reduced from 13 to 8.

Analysis shows that $\Phi$ is multimodal. Therefore, we use the multistart method to find a global minimum. The search when the increment declines to a certain value, corresponding to specified precision in determining the parameters $10^{-7}$ for the linear parameters and $10^{-8}$ for the angular parameters. On average, the target function takes the approximate value $\Phi \cong 10^{-17}$. In most cases, the global minimum may be localized by correct choice of the initial approximation. Such an initial approximation is the solution of Eq. (1) when $\lambda = 0$.

In calculating the optimal adjustment, we find that, in most cases, the fluctuation of the shaping trajectory in the vertical plane may be minimized to the required value, while the fluctuation in the horizontal plane is less susceptible to regulation. Research shows that the recommended geometric precision in the adjustment of a centerless superfinishing machine is 10″ for the skew angle and 0.1 mm for the interaxial distance.

Numerical experiments show that, in a superfinishing machine with rollers in the form of single-cavity

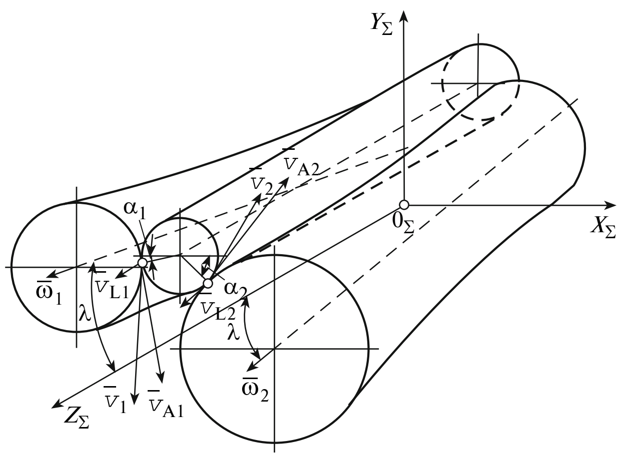


**Fig. 2.** Speeds in centerless superfinishing.

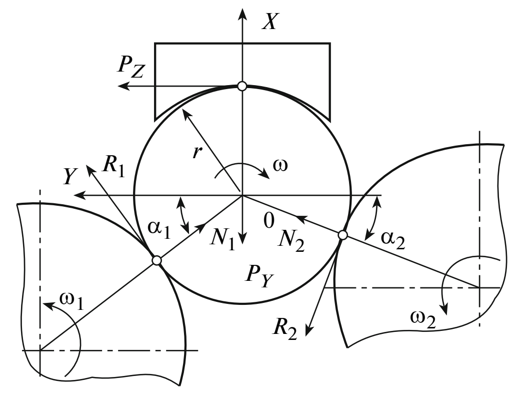


**Fig. 3.** Forces in centerless superfinishing.

hyperboloids, the trajectory of cylindrical blanks is close to the arc of a circle. Thus, such rollers may be used in machining cambered surfaces. In many cases, the laborious manufacture of strictly profiled rollers is unnecessary. This method is expedient when cambered surfaces whose profile radius is more than 3000 mm are manufactured from an initially cylindrical blank (diameter 10–50 mm). The limitation on the minimum arc radius of the shaping trajectory is due to design constraints of the existing equipment: $2\lambda$ must be no more than 9°.

We now consider the transfer of motion from the rollers to the blank (Fig. 2). The configuration of the rollers must be such that the component of the azimuthal-velocity vector along the blank's trajectory is directed from the input to the machining zone to its output. As a rule, the rollers turn in the same direction, with equal and constant angular velocities $\overline{\omega_1}$ and $\overline{\omega_2}$.

Since the azimuthal velocities $v_1$ and $v_2$ of the rollers do not agree in different cross sections, in the general case, only one roller will be the driving roller. In the absence of slip, the azimuthal velocity of the blank is equal to that of the roller at the contact point. On the assumption that the driving roller has already been determined by solution of the force problem, we will proceed to the analysis of the blank's motion.

The roller velocity at contact with the blank is calculated as

$$\bar{v} = \bar{\omega}\bar{R}, \tag{3}$$

where $\bar{\omega}$ is the roller's angular velocity; $\bar{R}$ is the roller's radius vector at the contact point.

From Eq. (3), we may obtain expressions for the blank's azimuthal supply velocity

$$v_{az} = R\omega\sqrt{\cos^2\alpha\cos^2\lambda + \sin^2\alpha}; \tag{4}$$

and its longitudinal supply velocity

$$v_{lo} = R\omega\cos\alpha\sin\lambda. \tag{5}$$

In Eqs. (4) and (5), $R$, $\omega$, and $\alpha$ correspond to the driving roller.

Research shows that the longitudinal velocity $v_{lo}$ varies monotonically from minimum to maximum over the whole machining length, with the recommended geometric adjustment of the machine [5]. With increase in the skew angle of the roller axe, both the absolute velocities and their difference at the edges of the machining zone will vary. Thus, we cannot ensure uniform longitudinal motion of an individual blank. Accordingly, retarding plates at the output from the machining zone are used in practice to create a continuous flux of blanks.

Now consider the forces in the blank's cross section (Fig. 3). The forces ensuring contact between the blank, the two basing rollers, and the abrasive bar must ensure stable rotation of the blank. The stability of the contact is determined by the frictional conditions at contact points of the blank and rollers, which, in turn, depend on the forces that arise in centerless superfinishing [6].

The required contact conditions may be obtained from the equilibrium equations in coordinate system $XOY$

$$\left.\begin{aligned} &\Sigma X = R_1\cos\alpha_1 - R_2\cos\alpha_2 + N_1\sin\alpha_1 \\ &+ N_2\sin\alpha_2 - P_Y = 0; \\ &\Sigma Y = R_1\sin\alpha_1 + R_2\sin\alpha_2 - N_1\cos\alpha_1 \\ &+ N_2\cos\alpha_2 + P_Z = 0; \\ &\Sigma M_0 = (R_1 + R_2 - P_Z)r = 0, \end{aligned}\right\} \tag{6}$$

where $N_1$, $N_2$ are the blank's normal reaction forces; $R_1$, $R_2$ are the frictional forces of the blank with the rollers; $P_Y$ is the force applied to the abrasive bar (the radial component of the cutting force); $P_Z$ is the tangential component of the cutting force; $r$ is the radius of the blank.

In superfinishing, the tangential component $P_Z$ depends on the force $P_Y$ and may be expressed as $P_Z =$

$kP_Y$, where $k$ is the cutting coefficient [1]. The frictional forces ($R_1$, $R_2$) are expressed in terms of the normal reaction ($N_1$, $N_2$) and frictional coefficients ($f_1$,$f_2$) of the blank with the left and right rollers, respectively. This is valid for steady motion or transition from rest to rotation. Assuming equal frictional coefficients of the rollers ($f_1 \approx f_2 = f$), we find that $R_1 = N_1 f_1$, $R_2 = N_2 f$.

For rotation of the blank, the total torque due to the frictional forces must exceed the torque due to the cutting force. Then the rotation of the blank is determined by the positive value of the total torque $\Sigma M_0$

$$F(N_1 + N_2) > kP_Y. \qquad (7)$$

If Eq. (7) does not hold, the blank stops. This corresponds to simultaneous slip of the blank relative to two rotors, which are turned by the machine's drive, with sufficient power margin. The blank is then immobile relative to the abrasive bar fixed in coordinate system *XOY*. In the general case, the blank, as it rotates, will slip over one of the rollers, in view of the difference in their azimuthal velocities at different contact points.

Solving Eq. (6) so as to eliminate the unknown forces, we obtain an invariant boundary condition for forced contact

$$(S + Q/k + T)f^2 + (Q - T/k)f + S = 0, \qquad (8)$$

where $S = \sin(\alpha_1 + \alpha_2)$; $Q = \sin\alpha_1 - \sin\alpha_2$; $T = \cos\alpha_1 + \cos\alpha_2$.

Thus, the stability of forced contact in centerless superfinishing depends on the geometric adjustment of the machine (specified by the angles $\alpha_1$, $\alpha_2$), the frictional coefficient $f$ at the points of contact–roller contact, and the cutting coefficient $k$. Analysis shows that the forced contact is nonlinear relative to $\alpha_1$, $\alpha_1$ (and correspondingly $\lambda$, $\nu$). With increase in $f$ and decrease in $k$, the range of permissible contact angles expands: for example, $\alpha_1 + \alpha_2 \approx 120°$ when $f = 0.17$ and $k = 0.35$.

In investigating the kinematic parameters, we need to identify the driving roller. This entails determining the normal reaction forces $N_1$ and $N_2$ from Eq. (6) and comparing them. The roller with the larger $N$ value will be the driving roller. Experiments for rollers with strictly calculated profiles show that one roller will be the driving roller over the whole machining length. In fact, $N_1$ always exceeds $N_2$ (Fig. 3), since the projection of force $P_Z$ onto the normal to the surfaces presses the blank against the left roller.

In the general case, the parameters $\lambda$ and $\nu$ of the superfinishing machine are optimized by minimizing the target function $Q(\lambda, \nu)$ obtained from the conditional target function $G(\lambda, \nu)$ by taking account of the structural, kinematic, and force constraints represented by the penalty function $U(\lambda, \nu)$: $Q(\lambda, \nu) = G(\lambda, \nu) + U(\lambda, \nu)$.

The function $G(\lambda, \nu)$ is the vector of shaping-trajectory deviations formed algorithmically as the difference between the maximum and minimum trajectory deviations in the vertical plane: $G(\Phi, \lambda, \nu) = \{b_{max} - b_{min}\} \longrightarrow \min$, where $\Phi$ is determined from Eq. (2).

The penalty function $U(\lambda, \nu)$ takes account of the disruption of constraints of the form $\varphi_j(\lambda, \nu) = 0$ and $\psi_i(\lambda, \nu) \geq 0$

$$U(\lambda, \nu) = r_1\varphi^2(\lambda, \nu) + r_2\sum_{i=1}^{2}\min\{\psi_i(\lambda, \nu)\}^2, \qquad (9)$$

where the coefficients $r_1$, $r_2$ are selected so as to ensure precise and economical calculations.

In Eq. (9), the structural and kinematic constraints take the form $\psi_i(\lambda, \nu) \geq 0$ and are linear; the force constraints take the form $\varphi(\lambda, \nu) = 0$ and are specified as the boundary conditions in Eq. (8). The relation between ($\lambda$, $\nu$) and the contact angles ($\alpha_1$, $\alpha_2$) of the blank with the rollers is determined by solving Eq. (1).

The experiments show that, in most cases, the deviation of the shaping trajectory in the vertical plane may be minimized to 0.001–0.003 mm. The fluctuation in the horizontal plane is less susceptible to regulation and, in some cases, minimization of the deviation in the vertical plane may increase the fluctuation in the horizontal plane. Accordingly, we need to correct the position of the abrasive bars relative to the blank's trajectory. To this end, we propose adjustment in which the roller system is rotated relative to the carriage of the superfinishing machine [7].

The proposed method increases the machining precision and the efficiency of adjustment and expands the scope for centerless superfinishing machines in machining cylindrical and cambered surfaces of bearing components.

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