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More Precise Superfinishing by Means of Statistical Modeling

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Superfinishing is the final stage in the manufacture high-precision parts that take the form of solids of revolution. The main goal of superfinishing is the increase the precision of the part's cross-sectional profile. It is assessed in terms of noncircularity—in particular, oval distortion, faceting, and undulation of the surface. Production experience shows that faceting and undulation may be effectively reduced in most cases, whereas it is much more difficult to reduce oval distortion.

In centerless superfinishing, stable reduction of oval distortion is impossible for the following reasons.

(1) Centerless basing tends to retain the shape of the blank from preceding operations. Therefore, much of the resultant error consists of basing error, with partial copying of the initial shape errors and the formation of new errors.

(2) In a batch of blanks, the shape fluctuations are stochastic, and it is not always possible to identify one or two dominant harmonics of the profile. Therefore, setup of the machine tool on the basis of determinate models only ensures precision in the first approximation.

Thus, further improvement in the precision of centerless superfinishing entails creating statistical models of shaping for batches of blanks and developing methods of machine-tool setup on that basis.

The mathematical model of basing in centerless superfinishing was considered in [1]. However, certain simplifications in the formulation of the problem introduced considerable error in the calculation. In addition, random initial phases and correlations between the amplitudes of the harmonics significantly complicate the problem and call for other methods of solution.

In the present work, we adopt a statistical modeling method—the Monte-Carlo method [2]. In machine-tool setup so as to minimize the basing error, this method involves simulation of the stochastic input data (the shape fluctuations of the blanks); repeated implementation of the analytical basing model; and the derivation of probabilistic characteristics whose numerical values agree with the results of determinate solution. In the end, we obtain a series of particular values of the basing error, whose statistical analysis reveals the influence of machine-tool setup on the machining precision for the batch of blanks.

The precision of the cross-sectional profile is assessed in terms of the noncircularity, which indicates how much the actual profile differs from some basic circle. Most often, the basic circle adopted is the mean circle corresponding to the periphery of the part's cross section according to the least-squares method [3]. Then the blank's cross section with shape fluctuations may be described by a trigonometric polynomial in polar coordinates [4]

$$r = r_0 + \sum_{n=2}^{p} a_n \cos(n\varphi - \varphi_n), \quad (1)$$

where r_0 is the radius of the blank's cross section; n is the angular frequency (the number of the harmonic); a_n is the amplitude of harmonic n; φ is the polar angle; φ_n is the initial phase of harmonic n; p is the maximum number of harmonics.

In centerless superfinishing, the blank is based at two rollers, whose cross sections may be represented by circles (radii R_1 and R_2) with centers at points A_1 and A_2 (Fig. 1). The position of these circles relative the origin of the coordinate system (XOY) is uniquely defined by contact angles α_1 and α_2.

By definition, the basing error is the deviation of the blank's actual position from its required position. In this case, the required position is the position of a

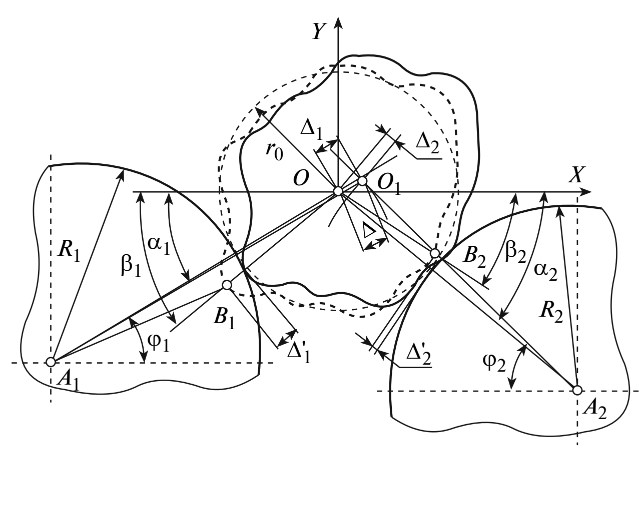

Fig. 1. Calculation of the basing error.

geometrically precise blank with some diameter, whose center O is at the origin of the coordinate system used to define the angles α_1 and α_2.

Thus, the problem reduces to determining the position of the blank's center O_1 relative to the coordinate origin O. To this end, we find the contact points of the blanks with the rollers when it is in the required position (shown by the dashed curve in Fig. 1). Obviously, these will be the points B_1 and B_2 closest to the basing rollers. We now establish the gap between the rollers and the blank in the initial position, in terms of its magnitude Δ' and the polar angle β

$$\left.\begin{aligned} \Delta_1'(\beta_1) &= \{r(\beta_1+180°)-r_1(\beta_1)\}\longrightarrow \max;\\ \Delta_2'(\beta_2) &= \{r(360°-\beta_2)-r_2(\beta_2)\}\longrightarrow \max,\end{aligned}\right\} \qquad (2)$$

where r_1 and r_2 are the radius vectors of the cross sections of the left and right rollers, respectively.

The cross section of the left roller is a circle with the displaced center A_1 relative to the origin of the polar coordinate system

$$r_1^2-2r_1(R_1+r_0)\cos(\varphi-\alpha_1)+r_0(2R_1+r_0) = 0. \quad (3)$$

Equation (3) has two roots. The root with a minus sign ahead of the square root is physically significant

$$\begin{aligned} r_1 &= (R_1+r_0)\cos(\varphi-\alpha_1)\\ &-\sqrt{(R_1+r_0)^2\cos^2(\varphi-\alpha_1)-r_0(r_0+2R_1)}.\end{aligned} \quad (4)$$

We describe the cross section of the right roller (radius vector r_2) analogously, in the polar coordinate system.

We assume that, in basing, the blank described by Eq. (1) tends to occupy a stable position at the two rollers. The blank has constant point contact with the two rollers and moves from its rated position by rolling successively over each roller. Then the projection Δ_1 of displacement Δ_1' onto the direction OA_1 is found from triangle OA_1B_1 according to cosine theory (Fig. 1)

$$\begin{aligned}&(R_1-\Delta_1)^2\\ &= r_1^2+(R_1+r_0)^2-2r_1(R_1+r_1)\cos(\beta_1-\alpha_1).\end{aligned}$$

Hence

$$\Delta_1 = R_1-\sqrt{r_1^2+(R_1+r_0)^2-2r_1(R_1+r_1)\cos(\beta_1-\alpha_1)}.$$

The projection Δ_2 of displacement Δ_2' onto the direction OA_2 is found analogously from triangle OA_2B_2

$$\Delta_2 = R_2-\sqrt{r_2^2+(R_2+r_0)^2-2r_2(R_2+r_2)\cos(\beta_2-\alpha_2)}.$$

As a result, the distance from the center of the blank O_1 to the center of the left roller is $A_1O_1 = R_1 + r_0 + \Delta_1$, and the distance to the center of the right roller is $A_2O_1 = R_2 + r_0 + \Delta_2$. To reach position O_1, the center of the blank moves successively over the left roller along an arc of radius A_1O_1 and over the right roller along an arc of radius A_2O_1. The intersection of these circles will determine the coordinates of point O_1 in the form of projections onto the X and Y axes

$$\left.\begin{aligned} &-(R_1+r_0)\cos\alpha_1+(R_1+r_0+\Delta_1)\cos\varphi_1\\ &= (R_2+r_0)\cos\alpha_2-(R_2+r_0+\Delta_2)\cos\varphi_2;\\ &-(R_1+r_0)\sin\alpha_1+(R_1+r_0+\Delta_1)\sin\varphi_1\\ &= (R_2+r_0)\sin\alpha_2+(R_2+r_0+\Delta_2)\sin\varphi_2,\end{aligned}\right\} \quad (5)$$

where φ_1 and φ_2 are the inclinations of vectors A_1O_1 and A_2O_1 with respect to the X axis.

In Eq. (5), the first terms on the left and right sides are the projections of the centers of the roller peripheries, while the second terms are the projections of the radii of the trajectories traveled by the center of the blank.

Solution of Eq. (5) yields formulas for the basing error

$$\left.\begin{aligned} \Delta &= \sqrt{(R_1+r_0+\Delta_1)^2+(R_1+r_0)^2-2(R_1+r_0+\Delta_1)(R_1+r_0)\cos(\alpha_1-\varphi_1)};\\ \varphi_1 &= \arccos\left[\frac{-ac-\sqrt{(ac)^2-(c^2-b^2)(a^2+b^2)}}{2(a^2+b^2)}\right],\end{aligned}\right\} \quad (6)$$

where $a = (R_2 + r_0)\cos\alpha_2 + (R_1 + r_0)\cos\alpha_1$; $b = (R_2 + r_0)\sin\alpha_2 - (R_1 + r_0)\sin\alpha_1$; $c = \dfrac{(R_2+r_0+\Delta_2)^2-(a^2+b^2)-(R_1+r_0+\Delta_1)^2}{2(R_1+r_0+\Delta_1)}$.

In centerless superfinishing, the blank rotates continuously, while the center O_1 of its cross section constantly changes its position. Therefore, the basing error is a variable. As a result of one rotation, point O_1 describes some closed contour, which characterizes the basing error.

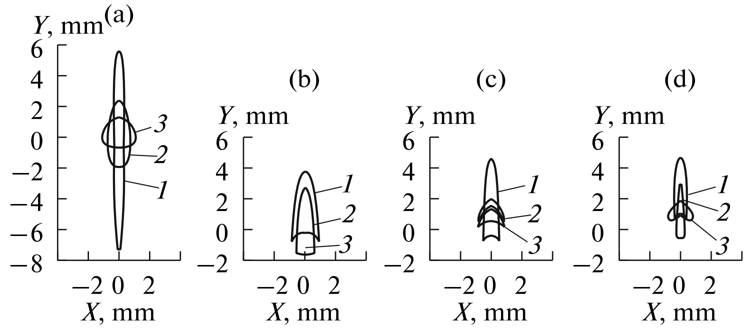

Fig. 2. Trajectories of the blank's center: (a) second harmonic; (b) third harmonic; (c) fourth harmonic; (d) fifth harmonic; (*1*) $\alpha = 20°$; (*2*) $\alpha = 40°$; (*3*) $\alpha = 60°$.

Basing in terms of the mean radius of the blank's center in a single rotation was proposed in [5]

$$K = \frac{1}{k}\sum_{i=1}^{k}\Delta(\varphi_i), \tag{7}$$

where φ_i is the blank's angle of rotation; k is the number of calculation points on the trajectory.

On the basis of computer simulation by Eqs. (1)–(7), we plot the trajectory of the blank's center as a function of the blank–roller contact angles (Fig. 2). The mean radius of the blank $r_0 = 8$ mm; the shape fluctuations take the form of second, third, fourth, and fifth harmonics, with amplitudes $a_2 = a_3 = a_4 = a_5 = 1$ μm. The variable selected is the total setup angle $\alpha = \alpha_1 + \alpha_2$, since each angle α_1 and α_2 only influences the coordinate origin in rotation of the blank. The roller radii are $R_1 = R_2 = 62.5$ mm, while the positions of their centers A_1 and A_2 are uniquely determined by the parameters α_1, α_2, and r_0.

The trajectory of each point of the blank's cross section takes the form of a curve congruent with the trajectory of the center. Hence, the trajectory of the blank's center determines the shape and fluctuation of the cross section. In Fig. 2, the trajectories are symmetric with respect to the Y axis, since, for the sake of simplicity, the null phases of the harmonics are assumed to be zero, while the contact angles α_1 and α_2 are equal ($\alpha_1 = \alpha_2$). Because the shape fluctuations are periodic, the trajectories of the blank's center repeat n times in a single rotation, where n is the number of the given harmonic.

Analysis of characteristic trajectories of the blank's center in centerless superfinishing show that the trajectory is extended along the Y axis at small α, but extended along the X axis at large α. With a linear trajectory—for example, for the third harmonic with $\alpha = 60°$—the blank's center passes through this line twice. For each harmonic, there is some optimal α value, which corresponds to the shortest trajectory.

Generally, in setup, oval and three-sided trajectories predominate, on account of the appearance of the corresponding errors after centerless superfinishing. In harmonic analysis, these curves take the form of a harmonic spectrum; in the first approximation, oval distortion may be described by the second harmonic. Numerical experiments do not reveal strict relations between the shape of the trajectory and the α values for individual harmonics.

Simulation permits the calculation of K as a function of α (Fig. 3). For the second harmonic (when $\alpha = 10°$ and 15°) and for the third harmonic (when $\alpha = 10°$), the blank enters the gap between the rollers (in different positions as it rotates). Therefore, K is not calculated for these cases.

Analysis of K shows that, for the second, third, fourth, and fifth harmonics, the value $\alpha = 60°$ is optimal within the range $\alpha = 10°–60°$. For the given examples, $K = 1–4.5$, which indicates the reproduction of basing errors and the creation of new errors. With decrease in the ratio of the roller radii R_1 and R_2 and the blank's radius r_0, some decrease in K is observed. Further research shows that the optimal α value is in the range 60–100°. However, such α values are not feasible in centerless superfinishing, on account of the forces required [6].

Setup of the centerless superfinishing machine is based on statistical assessment of K by the Monte-Carlo method for a certain batch of blanks. The basic

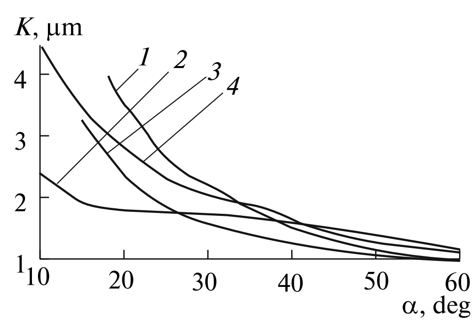

Fig. 3. Dependence of K on the setup angle α when $n = 2$ (*1*), 3 (*2*), 4 (*3*), and 5 (*4*); n is the number of the harmonic.

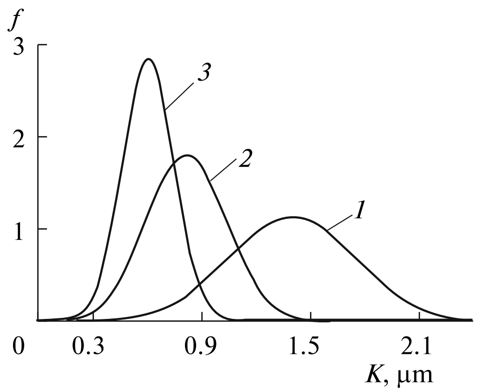

Fig. 4. Dependence of the probability density *f* on *K* when $\alpha = 15°$ (*1*), 30° (*2*), and 45° (*3*).

principles of this approach are as follows: simulation of the stochastic input data (shape fluctuations of the blank); repeated implementation of the analytical model of basing; and statistical analysis of the results of the determinate problem. Data regarding the shape errors are obtained experimentally; the distributions are calculated from statistical formulas. If the number of implementations is sufficiently large, the results will be statistically stable and may be regarded as estimates of the machining parameters, with satisfactory accuracy.

The initial data for the simulation are the parameters of the blank (radius r_0, number of harmonics n, distribution parameters, and limiting values of the amplitudes and initial phases of the harmonics); the setup parameters of the machine tool (basing angles α_1 and α_2, roller radii R_1 and R_2); and the number m of blanks in the batch [7].

To form a database regarding the distribution of the shape fluctuations for batches of blanks, experiments are conducted at OAO Saratovskii Podshipnikovyi Zavod. Attention focuses on batches of bearing rollers machined successively on SASL-200 × 500 centerless grinding machines and then SZZ-3 centerless superfinishing machines. In the experiments, the tendency to reproduce preexisting shape errors in centerless superfinishing is confirmed. The results indicate that the amplitudes a of the harmonics are characterized by a Pearson distribution of the first type, and the initial phases φ according to an equiprobable distribution.

We generate random numbers with the specified distribution by the inversion method, which entails the formation of a sequence of random numbers uniformly distributed within the interval [0, 1] and their subsequent transformation to the required distributions. The experiments show that there are significant correlations between some amplitudes of the harmonics. In that case, we may use the method of deriving mutually correlated numbers with arbitrary distributions from [8]. The parameters of the Pearson distribution obtained by statistical analysis of the experimental data are not integers; therefore, we employ the generation method proposed in [9].

On the basis of experimental data, we simulate K for centerless superfinishing in a batch of 200 blanks. Statistical analysis of the results shows that K is best described by a normal distribution. The probability-density function is uniquely determined by two parameters: the mathematical expectation m and mean square deviation σ. In Fig. 4, we plot the probability density f for setup with $\alpha = 15°$, 30°, and 45°. In all cases, the asymmetry is similar, while the excess index is three or more. For the worst value $\alpha = 15°$, the mathematical expectation of the basing error is 1.4 μm, while the mean square deviation is 0.351 μm. For the best value $\alpha = 45°$, these values are reduced by factors of around 2.3 and 2.5, respectively. With constraints on the maximum α value, the decrease in the mathematical expectation and mean square deviation is around 50%.

Comparison of the mathematical expectation of K and the noncircularity of the parts shows that optimal setup corresponds to active correction of the cross section. In addition, the lowest mathematical expectation corresponds to the least mean square deviation. Thus, setup of the superfinishing machine for a specific batch of blanks on the basis of a statistical model is more effective than setup that takes no account of the initial data regarding the blank's shape errors.

On the basis of our results, we may formulate general recommendations for more precise shaping of blanks in centerless superfinishing: setup must ensure maximum contact angles of the blanks with the basing rollers over the whole machining length; the kinematic and mechanical parameters of the process and the design capabilities of the superfinishing machine serve as constraints.

REFERENCES

1. Brzhozovskii, B.M., Zakharov, O.V., and Pogorazdov, V.V., Cross-Sectional Shaping of Parts in Centerless Superfinishing, *Stanki Instrum.*, 2002, no. 12, pp. 24–27.
2. Sovetov, B.Ya. *Modelirovanie sistem: Uchebnik dlya vuzov* (System Simulation: A University Textbook), Moscow: Vysshaya Shkola, 1998.
3. Avdulov, A.N., *Kontrol' i otsenka kruglosti detalei mashin* (Monitoring and Assessing the Noncircularity of Machine Parts), Moscow: Izd. Standartov, 1974.
4. Zakharov, O.V., Pogorazdov, V.V., and Kochetkov, A.V., Methodological Principles in the Harmonic Analysis of Cyclograms, *Metrologiya*, 2004, no. 6, pp. 3–10.
5. Zakharov, O.V., *Minimizatsiya pogreshnostei formoobrazovanii pri bestsentrovoi abrazivnoi obrabotke* (Minimi-

zation of Shaping Errors in Centerless Abrasive Machining), Saratov: SGTU, 2006.

6. Brzhozovskii, B.M., Zakharov, O.V. and Balaev, A.F., Kinematic and Mechanical Aspects of Centerless Superfinishing, *Stanki Instrum.*, 2006, no. 11, pp. 2–5.
7. Zakharov, O.V. and Brzhozovskii, B.M., Ensuring Precision in Centerless Abrasive Machining of a Batch of Blanks, on the Basis of Monte Carlo Simulation, *Vestn. SGTU*, 2006, no. 4, issue 1, pp. 12–17.
8. Labutin, S.A. and Putin, M.V., Summation of Random Measurement Errors and Monte Carlo Analysis of Indirect Measurement Errors, *Izmerit. Tekhn.*, 2000, no. 11, pp. 6–9.
9. Hastings, N. and Peacock, B., *Statistical Distributions: A Handbook for Students and Practitioners*, New York: Wiley, 1975.