ISSN 1068-798X, Russian Engineering Research, 2009, Vol. 29, No. 1, pp. 75–77. © Allerton Press, Inc., 2009.
Original Russian Text © O.V. Zakharov, 2008, published in STIN, 2008, No. 10, pp. 5–8.

Increasing the Accuracy of Centerless Grinding on Motionless Bearings

O. V. Zakharov

DOI: 10.3103/S1068798X09010183

In centerless grinding on motionless bearings, the greatest problem is to ensure accurate shape of the parts [1–3]. In particular, the manufacture of parts that deviate from circularity by less than 0.002 mm is a major challenge.

Noncircularity of the blank is due to disruption of the relative trajectory of the blank and tool and also to basing errors, as established in [4, 5]. The shape errors of precision parts include not only the traditional components (due to elastic displacement, thermal deformation, vibration, etc.) but also the inherited geometric error of the blank.

A determinate model of centerless basing on motionless bearings was proposed in [6, 7]. The basing-accuracy characteristic K was proposed as a vector of microdisplacements of the center of the blank as it rotates relative to the basing coordinate system. Practical experience with this model shows that the determinate approach considerably simplifies the description of shaping.

It is inexpedient to set up centerless grinding machines for a specific blank, but the shape of the blanks in a batch will vary randomly. It is not always possible to isolate the dominant harmonic in the shape error, since there will be several harmonics of relatively large amplitude. In addition, the presence of random initial phases and correlations between the amplitudes of the harmonics significantly complicates the problem.

Statistical simulation (the Monte Carlo method) may be used here [8]. In setting up a machine tool with minimum basing error, this approach involves simulation of the stochastic input data (shape deviations of the blanks) and multiple implementation of the analytical basing model.

Simulation yields probability characteristics whose numerical values are equal to the results of solving the determinate problem. Statistical analysis of a series of basing-error values permits the determination of the influence exerted on the machining accuracy of a batch of blanks by the setup parameters of the machine tool.

The corresponding algorithm is shown in Fig. 1. The initial data include the blank parameters BP (radius r_0 of the mean profile circumference; number p of harmonics; distribution parameters and boundaries of the ranges of the amplitudes a_n and initial phases φ_n of the harmonics), the setup parameters S of the machine tool (the angle α between the motionless bearings), and the number m of blanks in the batch. The variation of α in optimization is bounded on the basis of the machine-tool characteristics.

In the first stage, the geometric shape deviations in the batches of blanks are simulated. First, sequences of uniformly distributed numbers random z_i are generated for each blank j, as a function of the number of profile harmonics ($i = \overline{1, m}$). The values of z_i are converted to the required distribution law for the amplitude a_{ni} and initial phase φ_{ni} of each harmonic. As a result of summation, the profile r_j of the blank is formed.

The cross section of the blank with periodic shape deviations is described by a trigonometric polynomial of the form $r = r_0 + \sum_{n=2}^{p} a_n \sin(n\varphi - \varphi_n)$, where r is the radius vector; n is the number of the harmonic; φ is the polar angle; a_n and φ_n are the amplitude and initial phase of harmonic n.

The generation of random numbers with a specified distribution is based on the inversion method [8]. In this

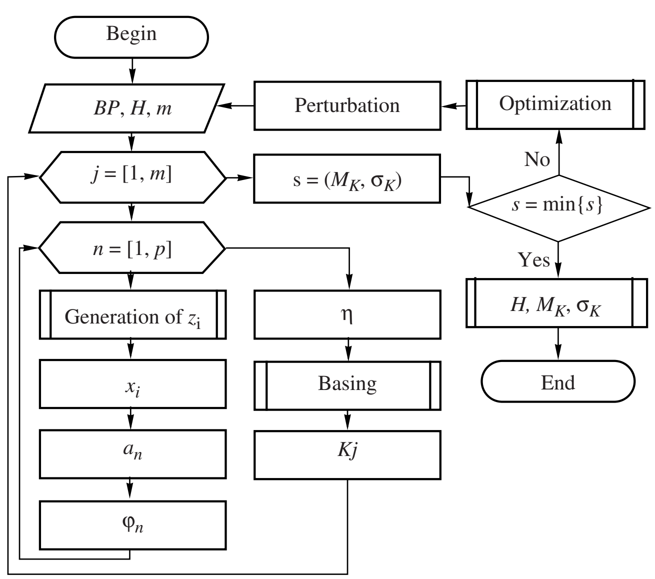

Fig. 1. Simulating algorithm for machine-tool setup (Monte Carlo method).

method, a sequence of random numbers z_i uniformly distributed in the interval [0, 1] is formed and then transformed: $x_i = F^{-1}(z_i)$, where $F^{-1}(z_i)$ is the inverse of the distribution function of the random quantity x_i.

Experiments show that the amplitudes a_n of the harmonics are best described by a β distribution, while the initial phases φ_n are best described by an equal-probability law.

The probability-density function of the β distribution takes the form $f(x_i) = \{[\Gamma(\eta + \mu)]/[\Gamma(\eta)\Gamma(\mu)]\} x_i^{n-1} \times (1 - x_i)^{\mu-1}$, where Γ is a gamma function; η and μ are the parameters of the β distribution. The distribution function of the initial phases of the harmonics takes the form $F(x_i) = (x_i - b)/(c - b)$, where b and c are the boundaries of variation of φ_n.

Experiments show that there are strong correlations between the amplitudes of some harmonics; these must be taken into account in simulation. Therefore, for random errors x_1 and x_2 with different distribution functions $F_1(x_1)$ and $F_2(x_2)$, mathematical expectations M_{x1} and M_{x2}, and mean square deviations σ_{x1} and σ_{x2}, it is expedient to switch to the random quantities z_1 and z_2 uniformly distributed in the interval [0, 1], by means of the transformations

$$z_1 = F_1\left(\frac{x_1 - M_{x1}}{\sigma_{x1}}\right); \quad z_2 = F_2\left(\frac{x_2 - M_{x2}}{\sigma_{x2}}\right).$$

The linear correlation coefficient is [9]

$$k = \frac{12}{m-1}\sum_{j=1}^{m}\left(z_{1j} - \frac{1}{2}\right)\left(z_{2j} - \frac{1}{2}\right),$$

where z_{1j} and z_{2j} are the same as z_i when $i = 1$ and 2 for a sample with $j = \overline{1, m}$.

This simplifies the generation of correlated random quantities with different distributions and eliminates the dependence of k on the form of these distributions.

Pairs of correlated random numbers with different distributions are obtained as follows [10]. In the first stage, three uncorrelated random numbers A, B, and C with a uniform distribution in the interval [0, 1] are generated. Then, a pair of correlated numbers (random numbers y_1 and y_2) is formed, as follows

$$\left.\begin{aligned} y_1 &= A\sqrt{|k^*|} + B\sqrt{1 - |k^*|}; \\ y_2 &= A\sqrt{|k^*|} + C\sqrt{1 - |k^*|}, \end{aligned}\right\} \qquad (1)$$

where k^* is the correlation coefficient of y_1 and y_2.

The correlation coefficients k and k^* are related as

$$|k^*| = |k| + 0.005086 + 0.01739\sin(6.3986|k| + 5.975). \qquad (2)$$

In general, y_1 and y_2 are distributed according to a symmetric trapezoidal law, with large s_1 and small s_2 sides of the trapezium. Then, the trapezoidal distribution is converted to a uniform distribution within the interval [0, 1]:

if $y_{1,2} > s_1/2$, then when $g = s_2 - y_{1,2}$

$$z_i = \begin{cases} 1 - 2g^2/(s_1^2 - s_2^2) \text{ when } |g - s_1/2| > s_2/2; \\ 1 - (s_2 - s_1 + 4g)/[2(s_1 + s_2)] \\ \text{when } |g - s_1/2| \le s_2/2; \end{cases}$$

if $y_{1,2} \le s_1/2$, then when $g = y_i$

$$z_i = \begin{cases} 2g^2/(s_1^2 - s_2^2) \text{ when } |g - s_1/2| > s_2/2; \\ (s_2 - s_1 + 4g)/[2(s_1 + s_2)] \\ \text{when } |g - s_1/2| \le s_2/2; \end{cases}$$

here $s_1 = \sqrt{|k^*|} + \sqrt{1 - |k^*|}$; $s_2 = s_1\left|1 - (2/s_1)\sqrt{|k^*|}\right|$.

Thus, to find the required correlation coefficient between z_1 and z_2, we need to specify $|k^*|$ from Eq. (2) when formulating the random numbers y_1 and y_2. On switching the parameter pairs A and B, A and C in Eq. (1), z_1 and z_2 will correspond to the correlation coefficient $1 - |k|$, which permits the formation of samples of two random numbers with correlation coefficient $\pm(1 - |k|)$ and arbitrary distribution.

For a uniform distribution, the random quantity z_i is converted to the random quantity x_i by the formula $x_i = 360z_i$.

The parameters η and μ of the β distribution, which are obtained by statistical analysis of experimental data, are not integers. Therefore, we use the generation method in [11]. We calculate $S_1 = z_1^{1/\eta}$ and $S_2 = z_2^{1/\mu}$. If $S_1 + S_2 \ge 1$, we take another pair of random numbers z_1 and z_2 and perform the same operation. If $S_1 + S_2 < 1$, then we take $x_i = S_1/(S_1 + S_2)$.

In the second stage, we calculate the basing error for each blank from the analytical model [7]. Then we find the basing characteristic K_j for each blank j in the batch.

The third stage is statistical analysis of the K values calculated for all the blanks in the batch. As a result, we obtain the mathematical expectation M_K and the mean square deviation σ_K. Then, we optimize the setup parameters H of the machine tool so as to minimize M_K and σ_K.

Analysis of the results obtained by the proposed simulation algorithm show (Fig. 2) that K is best described by a normal or log-normal distribution. (The log-normal distribution appears when there is a correlation between groups of harmonic amplitudes.) The probability-density function is uniquely determined by M_K and σ_K, which are selected as the parameters to be optimized.

Numerical experiments indicate that M_K and σ_K are a minimum at the point corresponding to a specific setup angle α. It follows from Fig. 2 that, for the given

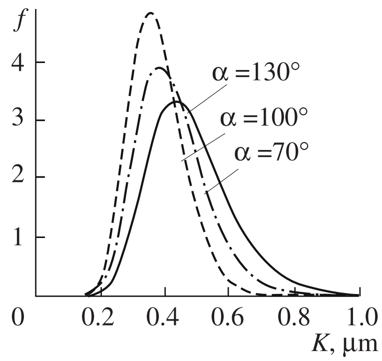

Fig. 2. Probability-density function f of K as a function of setup parameter α.

batch of blanks, the least rational value is $\alpha = 130°$. In this case, $M_K = 0.46$ μm and $\sigma_K = 0.267$ μm. The best value is $\alpha = 100°$, for which M_K is reduced by 21% and σ_K by 15%.

The proposed method is verified at OAO Saratovskii Podshipnikovyi Zavod in grinding 5-830900AE1.02 bearing rings on SWaAGL-50 machine tools. The tolerance for noncircularity of the annular channel is 0.0012 mm; the surface roughness $R_a = 0.32$ μm. In grinding the wheel and the blank move in the same direction, at speeds of 35 m/s and 35 m/min, respectively.

Optimal setup of the motionless bearings in the machine tool ($\alpha = 105°$, rather than 118°) reduces the mean noncircularity from 0.89 to 0.79 μm and the means square deviation from 0.195 to 0.165 μm. The rejection rate is reduced from 5.1% to 0.6%. The surface undulation and roughness of the parts complies with the technical requirements.

Research indicates that greater precision of centerless grinding on motionless bearings results from minimization of the basing errors, by optimal setup of the machine tool using statistical models.

REFERENCES

1. Bazrov, B.M., *Raschet tochnosti maschin na EVM* (Computer Calculation of Machine Precision), Moscow: Mashinostroenie, 1984.
2. *Tekhnologicheskie osnovy obespecheniya kachestva mashin* (Technological Principles of Machine Quality), Kolesnikov, K.S., Ed., Moscow: Mashinostroenie, 1990.
3. *Tekhnologicheskaya nasledstvennost v mashinostroitel'nom proizvodstve* (Technological Inheritance in Manufacturing), Dal'skii, A.M., Ed., Moscow: Izd. MAI, 2000.
4. Fil'kin, V.P. and Koltunov, I.B., *Progressivnye metody bestsentrovogo shlifovaniya* (Progressive Methods of Centerless Grinding), Moscow: Mashinostroenie, 1971.
5. Zakharov, O.V., *Minimizatsiya pogreshnostei formoobrazovaniya pri bestsentrovoi abrazivnoi obrabotke* (Minimizing the Shaping Errors in Centerless Abrasive Machining), Saratov: SGTU, 2006.
6. Zakharov, O.V. and Pogorazdov, V.V., Minimizing the Periodic Shaping Errors in Centerless Grinding on Motionless Bearings, *Stanki Instrum.,* 2006, no. 1, pp. 25–28.
7. Zakharov, O.V. and Gorshkov, V.V., Ensuring Accuracy in Centerless Grinding with Transverse Supply, *Avtomatizatsiya i Sovrem. Tekhnol.*, 2006, no. 10, pp. 11–15.
8. Sovetov, B.Ya. and Yakovlev, S.A., *Modelirovanie sistem* (Modeling Systems), Moscow: Vysshaya Shkola, 1998.
9. Tyurin, Yu.N. and Makarov, A.A., *Statisticheskii analiz dannykh na kompyutere* (Computerized Statistical Data Analysis), Moscow: Infra-M, 1998.
10. Labutin, S.A. and Putin, M.V., Summing Random Measurement Errors and Analysis of the Errors of Indirect Measurements by the Monte Carlo Method, *Izmerit. Tekhn.*, 2000, no. 11, pp. 6–9.
11. Hastings, N. and Peacock, B., *Statistical Distributions: A Handbook for Students and Practitioners*, New York: Wiley, 1975.