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ACCURACY OF CENTERING DURING MEASUREMENT BY ROUNDNESS GAUGES

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The effect of centering accuracy on the error for measurement of roundness deviation is considered. A method is proposed for processing the results of measurements based on harmonic analysis, compensating for a given error. Results are given for comparative analysis by the well-known and suggested methods in measuring deviation from roundness on the basis of Monte-Carlo statistical modelling.
Key words: *roundness plot, roundness deviation, centering, harmonic analysis.*

A significant proportion of machine components and mechanisms comprises a rotating body on whose dimensional accuracy and shape the quality of a machine as a whole depends. Therefore in technology considerable attention is devoted to monitoring the surface of these components, and this has determined the development of a considerable number of measurement methods, each of which cannot be universal but has its own rational field of application.

As a rule, in monitoring shape deviation in a cross section a complex criterion is used, i.e., deviation from roundness [1]. Currently, the required measurement accuracy by this criterion for precision components of machine and instrument building is only provided by roundness gauges with reference rotation of a spindle [2]. The most well-known instruments are those produced by Taylor Hobson (Great Britain) under the trademark Talyrond. The operational radius measurement error for them is 0.01 μm. The average roundness of a profile is used as a basis in analyzing quality.

The choice of centering accuracy should be considered fundamental, on which depend on one hand, the accuracy of measuring deviation from roundness, and on the other hand, measurement productivity. It is apparent that these requirements are in opposition to each other. According to recommendations in [3], the accuracy of centering should correspond to measurement accuracy. However, such severe requirements for centering accuracy lead to a manifold increase in the expenditure of time in order to accomplish it. In addition, with the use of manual centering by the operator so-called residual eccentricity always remains from the result of radial wobble.

For automation of the centering procedure, and in some cases also for processing the results of measurements (for example, with coordinate measurements [4]), the Spragg equation [3] is used extensively due to its simplicity:

$$R = \frac{1}{n}\sum_{j=1}^{n} r_j; \qquad x = \frac{2}{n}\sum_{j=1}^{n} x_j; \qquad y = \frac{2}{n}\sum_{j=1}^{n} y_j, \tag{1}$$

where R, x, and y are the radius and coordinates of the center of the central circle of the profile; r_j is the length of the radius-vector of the jth point of the profile; x_j, y_j are Cartesian coordinates of the jth point of the profile; n is the number of measured points of the profile.

The approximate nature of Eq. (1) was noted in [2, 4] that only gives good results with small eccentricities for the center of the central circle of the profile with respect to the center of the measuring system.

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Another approach to solving the problem includes compensation for the procedural error of centering in the stage of processing the results. Specialists of Taylor Hobson have suggested using harmonic analysis for this purpose. The essence of the method is that the results of harmonic analysis are used to find the analytical equivalent of article profile r in the form of a trigonometric polynomial

$$r = a_0 + \sum_{i=1}^{p} a_i \cos(i\varphi - \psi_i) = a_0 + \sum_{i=1}^{p} (b_i \sin i\varphi + c_i \cos i\varphi),$$

where a_0 is zero harmonic; a_i, ψ_i are the amplitude and initial phase for the ith harmonic; b_i, c_i are coefficients for the ith harmonic; p is the greatest number of harmonics; and φ is polar angle.

Coefficients of the trigonometric polynomial are calculated by the well-known Bessel equation [5]:

$$a_0 = \frac{1}{n}\sum_{j=1}^{n} r_j; \quad b_i = \frac{2}{n}\sum_{j=1}^{n} r_j \sin i\varphi_j; \quad c_i = \frac{2}{n}\sum_{j=1}^{n} r_j \cos i\varphi_j, \tag{2}$$

where r_j is length of the radius-vector of the jth point of the article profile; φ_j is polar angle of the jth point of the article profile; and n is the number of points on the article profile.

Considering that the zero harmonic a_0 is the radius if the central circle of the article profile, and the first harmonic (a_1, ψ_1) is eccentricity of the central circle with respect to the measuring coordinate system, compensation of eccentricity is accomplished by means of the first harmonic. However, practical application of this approach with measurement in roundness gauges with standard rotation of a spindle has not made it possible to do away with accurate centering.

Thus, a question arises regularly about the validity of using harmonic analysis with centering and measurement of deviation from roundness in roundness gauges. This question has already been raised in [6, 7] where excellent equations are shown describing the first harmonic and average roundness whose coordinate point measurement was carried out with some eccentricity.

We turn attention to the identical nature of (1) and (2) for the first harmonic, from which it follows that direct harmonic analysis and the Spragg equation lead to the same result acceptable for relatively small eccentricity of centers of the central circle by comparison with the measurement system. On the other hand, calculations provided subsequently for the effect of centering error on the accuracy of measuring deviation from roundness by means of harmonic analysis are also valid with use of the Spragg equation.

An equation was obtained in [8] whose characteristic includes establishing a connection of polar angles in the coordinate system for the article and the gauge making it possible to obtain a uniform angular arrangement of points on the profile with the presence of eccentricity:

$$r_1 = \sqrt{R^2 + e^2 + 2\,\mathrm{Re}\cos\{\varphi_1 - \psi + \arcsin[e\sin(\varphi_1 - \psi)/R]\}}, \tag{3}$$

where R is central circle radius; e, ψ are amplitude and initial phase of eccentricity; φ_1 is polar angle in the gauge system.

Harmonic analysis of expression (3) was carried out numerically since the solution in explicit form leads to cumbersome mathematical calculations. It has been established that the function prescribed by (3) is superposition of the first and even harmonics (p = 1, 2, 4, ...) with amplitudes decreasing with an increase in the order of the harmonic. It is possible to consider that only the amplitudes of the zero a_0, first a_1 and second a_2 harmonics are important. The first harmonic clearly determines the amplitude e and initial phase ψ of eccentricity, even though there is development of an eccentric circle and it differs from a sinusoid. Harmonic analysis has also shown that amplitude a_0 is less than radius R of the central circle by amplitude a_2.

Taking all of this into account, compensation of the procedural error should be accomplished by subtraction from the measured lengths of the radii-vectors of the points of profile r_1 of a correcting value Δr that is the sum of the first and second harmonics and the difference in amplitude of the second harmonic:

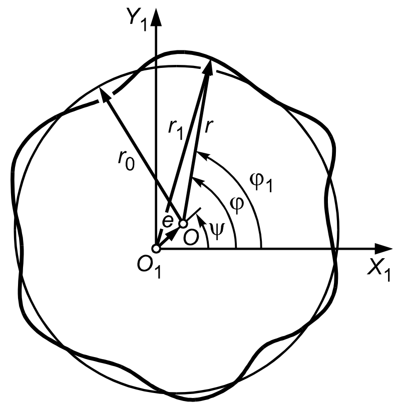

Fig. 1. Measurement scheme in a roundness gauge.

$$r = r_1 - \Delta r = r_1 - a_1\cos(\varphi - \psi_1) + a_2[1 - \cos(2\varphi - \psi_1)]. \quad (4)$$

In (4), the values a_1, ψ_1 are determined on the basis of harmonic analysis of the article profile according to (2), and the value of a_2 is determined by harmonic analysis of the eccentric central circle by (3).

Amplitude a_2 of the second harmonic is not connected linearly with amplitude a_1 of the first harmonic. Their ratio a_2/a_1 depends markedly on the ratio e/R. The maximum value of a_2/a_1 = 0.42 is achieved with eccentricity equal to the radius of the central circle. In the range $e/R \le 0.1$, the relationship a_2/a_1 is close to linear and this ratio is about 0.025. If the roundness plot is analyzed graphically, where instead of the article radius we consider the average radius of the record, commensurate to a greater degree with eccentricity, then the deviation will be several times greater.

The method in question, within which Eqs. (3) and (4) are used, has been approved and used in the method developed for measurement by roundness gauges [8]. We provide the results of comparative analysis of the accuracy for measuring deviation from roundness in relation to the accuracy of centering for the well-known and suggested procedure. Fifty bearing rollers 16 mm in diameter after centerless grinding were selected for the tests. Measurements were performed by means of Talyrond 73 (Taylor Hobson, Great Britain) instruments in the Saratov Ball Bearing Plant OAO. The measurement scheme for the roundness gauge is shown in Fig. 1.

The roundness plots obtained were subjected to harmonic analysis and statistical processing, and then modelled according to the Monte Carlo method. During harmonic analysis, harmonics from the 2nd to the 25th inclusively were considered, and harmonics with amplitudes less than 0.02 μm were not considered.

It was established that the amplitudes of harmonics are distributed by a Pearson rule of the first type, and deviation from roundness is distributed by a normal rule. Parameters of the Pearson distribution were found on the basis of parametric optimization where the efficiency function was the criterion χ^2. Similar results have been presented in [9] for an opening 1 mm in diameter prepared by cutting, sizing, and drilling.

Modelling of roundness plots with a specific previously valid distribution was performed by the well-known Monte Carlo statistical method. An inversion method was used for generation [10]. Correlation of amplitude and the initial phases for individual harmonics was not considered. The profile of a roundness picture was prescribed by 500 points (in accordance with the Talyrond 73 instrument data). Modelling was also performed for the random error of centering with radius e, distributed by a normal rule, and polar angle ψ, distributed by a rule of equal probabilities. Then the deviation from roundness was calculated by (1) for the well-known and by (2) for the suggested methods, and statistical processing was carried out.

Results are given in Table 1 for modelling with measurement of deviation from roundness for 500 roundness plots in the form of estimates of mathematical expectation $\bar{x}$ and mean square deviation σ obtained for three values of centering error. The results of calculations were rounded to the practically significant quantity 0.01 μm.

TABLE 1. Results of Modelling with Measurement of Deviation from Roundness

Centering error, mm	Deviation from roundness, μm					
	actual		by (2)		by (4)	
	$\bar{x}$	σ	$\bar{x}$	σ	$\bar{x}$	σ
0.01	1.11	0.14	1.11	0.14	1.11	0.14
0.05	1.11	0.14	1.15	0.16	1.11	0.14
0.20	1.11	0.14	3.02	0.23	1.11	0.14

Analysis of the results of modelling showed that with a centering error of 0.01 mm the well-known procedure gives a precise result, with 0.05 mm it is satisfactory, but with 0.2 mm it gives an unsatisfactory result. Calculation by the suggested procedure for deviation from roundness coincided with the actual value with an acceptable error of 0.01 μm with all values of centering error.

Thus, practical application of the procedure in question for processing the results of measurements in roundness plots makes it possible to reduce the requirement for centering accuracy by factors of 5–10 and thereby to increase measurement productivity. In conclusion, we note that this procedure for processing the results of measuring deviation from roundness in the presence of eccentricity may also be used for coordinate [11] and difference [12] measurement methods.

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